

# A MODEL FOR THE ESTIMATION OF USER DELAYS CAUSED BY PAVEMENT MAINTENANCE WORKS

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Some tools, like HDM for instance, have been developed in order to optimise the economic benefits given by road maintenance; they include models taking into account the main economic terms which are the user costs (time, safety, comfort, vehicle consumption), the road agency costs (road construction, maintenance and exploitation), and sometimes external costs like environmental damages or non renewable resource consumption. However, recent studies have shown that a full optimisation of road maintenance should take into account all the consequences of road works, including user costs at roadwork sites, which are key factors of the maintenance economic profitability, especially in European countries where traffic flows are often close to congestion levels. Reducing the frequency of road works, their duration and their impact on traffic flow (slowdowns and bottlenecks) will decrease the total delays experienced by users in their travels, and thus reduce the total economic cost of the maintenance policy. Such decisions may lead to an increase of the direct cost of works, but this can be accepted if the global balance is positive. The main problem in evaluating user delays caused by road works is that the response of flow condition is very sensitive to small changes in parameters like traffic demand and road capacity, even if these changes are very limited in time. In order to overcome this difficulty, a probabilistic model was developed, which can predict the instantaneous traffic, according to a normal distribution. The model parameters may be estimated through an analysis of traffic hourly counts performed during a series of days that can be distributed into different typical categories. This model will be used to calculate, on a given road network and for a given maintenance policy, the total volume of user delays associated to road maintenance operations, and so their cost and their impact on the total benefit assigned to the proposed solution.

**KEYWORDS:** TRAFFIC, MAINTENANCE, PAVEMENT, ROAD USER, WAITING TIME, PROBABILISTIC MODEL

## 1 OBJECTIVE

The cost benefit analysis, which has been used for a long time in the choice of road investments (new roads or improvements), is not yet of common use to optimise pavement maintenance strategies, at least in the industrialised countries. The problem stems from the need to “monetarise” all the costs and benefits resulting from the considered works, either direct or indirect, immediate or future, undergone by the public power, the user or the population in general...

But, if the real expenditure (realisation of works, fuel consumption, material cost of the accidents) can be established in an objective way, other costs, like those of the personal injuries or the effects on environment, rest on assumptions or presuppositions which do not reach a global consensus. In France, an important work of formalisation and evaluation of these costs was carried out within the framework of the “Boiteux report” [1].

The need for progress and convergence on this subject at the European level led to the FORMAT (Fully Optimised Road Maintenance) contract [2]. The LCPC takes an active role in this project, and particularly leads the part related to the evaluation of delays undergone by the users as a consequence of maintenance works.

The paragraphs which follow describe part of the work completed in 2002 within the framework of FORMAT, and more especially the probabilistic evaluation of waiting time. That consisted in determining, at any moment of one day, the probability that traffic flow reaches a given level; the comparison between this value and the residual capacity of the road under works then make it possible to calculate the probable number of hours lost by the users, and then the mathematical expectancy of this number.

## 2 THE STAKES

In order to evaluate the magnitude of the problem, let us take an example: on a dual carriageway road, where speed is limited to 110 km/h and where traffic flow amounts to 20,000 vehicles per day, a surfacing renewal must take place on 5 km during 10 days; this requires the closure of one carriageway, the other being exploited in both directions, at a 70 km/h speed limit. The direct cost of works is estimated at 30 €/m<sup>2</sup>, that is to say 1,2 M€ in total.

If traffic flows freely, the main effect will be that of speed limit, which amounts to 0.025 hr (one minute thirty) per vehicle, and thus 5 000 v.hr (vehicle-hours) for the whole works; at a rate of 13.7 € per v.hr (value recommended by the Boiteux Report for long distance trips), that yields to 68,500 €, which represents already 6 % of the cost of works.

Under more difficult conditions (where half of the vehicles would undergo a 15 minutes waiting, and would travel through the work site at 30 km/h), the total amounts to 12,500 v.hr for waiting and 19,000 v.hr for speed reduction, or a total of 430,000 € according to the preceding scale, what is more than the third of the direct cost. And yet this calculation ignores the consequences for the lorries, safety, fuel consumption and environment.

There are of course even more serious cases, but also palliative solutions (diversion routes, night-working, public information, incentives to use other means of transport) which can be treated in a cost benefit analysis, but all that exceeds our purpose...

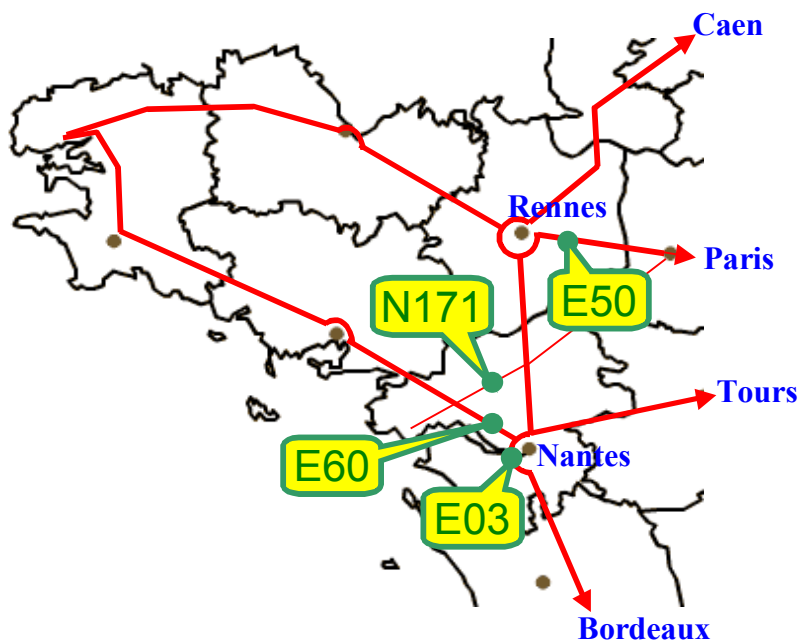
### 3 TRAFFIC MODEL

#### 3.1 Available counts

SIREDO (Computerised System of Data Collection) stations [3], distributed on the French national road network, ensure uninterrupted counts by one hour periods. Moreover, on certain stations, the number of lorries, the axle weights and the speeds are recorded. On prior request, exploitations such as counting per six minute periods can be obtained, but no retroactive request is possible, because the unit data are not recorded.

The count files were provided by the CETE (Centre d'Études Techniques de l'Équipement) de l'Ouest. Four stations were selected, corresponding to different levels and types of traffic:

1. A two-lane road in a rural zone, bearing a medium-heavy traffic (N 171).
2. A road bearing a strong interregional traffic, plus that resulting from the relations between two large cities and the access to beaches (E60).
3. A Paris to provinces route, with strong seasonal incidence (E50).
4. A regional metropolis ringroad (E03).



**Figure 1 – Localisation of counting stations**

These roads, except the first one, are exploited as expressways with two dual lane carriageways.

Only “all vehicles” counts were exploited here, because the lorry data were not available for all treated cases. The considered periods are:

- For hourly counts, the whole 2000 and 2001 years;
- For sharp counts (per six minutes), the period from June 20<sup>th</sup> to 29<sup>th</sup> 2002.

### 3.2 The deterministic model of traffic forecast

A first model trial consisted in predicting the traffic flow of the 24 one-hour periods from a limited number of counts: it was thus shown that counts on four hours was enough to reconstitute with a good precision all 24 hours. But this principle presented two disadvantages: first, the hours having the best predictive capacity were not the same according to the considered day and station, and then this model ignored the variations inside the hourly periods.

Then came the idea of a continuous function calculable from a limited number of parameters, and the visual aspect of the curves gave the way to be followed: for the various stations one can almost always see two traffic peaks, looking like a “bell-shape” curve, whose equation is identical to that of the probability density of the normal distribution law:

$$T(t) = \frac{a}{\sqrt{2\pi}s} \exp(-(t-m)^2 / 2s^2)$$

where:

- $T$  is the predicted value of the traffic flow,
- $t$  is the time, expressed in decimal hours as from zero hour for the day considered,
- $a$  represents the cumulated number of vehicles included in a peak,
- $m$  is the mean value, that is to say the value of  $t$  which maximises the function,
- $s$  is the standard deviation, which accounts for the sharpness of the peak.

In order to take into account the effect of peaks before zero hour and beyond 24 hours, the value of  $t-m$  was replaced in calculations by  $t-m-24$  if  $t-m > 12$  and  $t-m+24$  if  $t-m < -12$ .

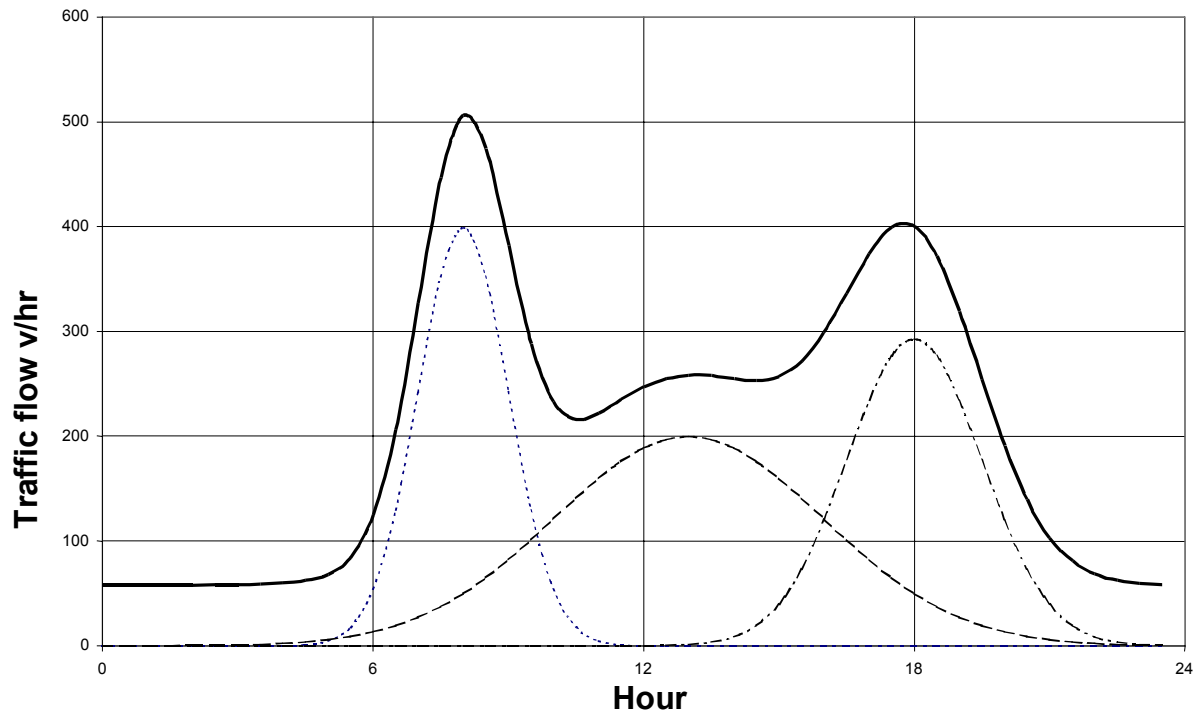
The StatGraphics-Plus<sup>®</sup> for Windows<sup>®</sup> software, version 4.1, was used to carry out a nonlinear regression according to this model, for each station and each day for which counts per six minutes were available.

It quickly appeared that convergence was not always obtained, particularly if the period ranging between the two peaks presented a third maximum or a flat zone at a high level: it was thus advisable to model this intermediate period, and adding third bell-shaped curve answered the problem quite well.

This gives the formula:

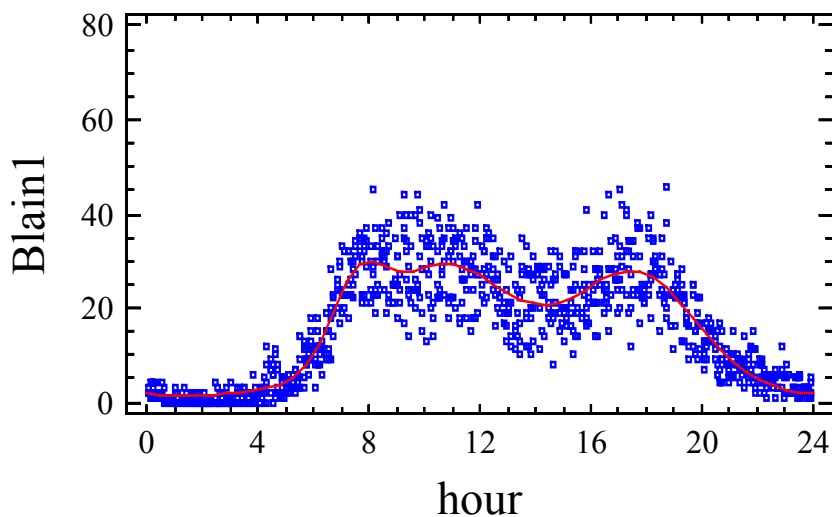
$$T(t) = a_0 + \sum_{i=1}^{i=3} \frac{a_i}{\sqrt{2\pi}s_i} \exp(-(t-m_i)^2 / 2s_i^2)$$

The predicted value of the traffic can thus be calculated for any time of the day, by using ten parameters (the constant  $a_0$  and the three  $a_i$ ,  $m_i$  and  $s_i$ ). It is called the “deterministic component” of the model. Figure 2 shows the principle of such a calculation, and figure 3 a result of nonlinear regression.



**Figure 2 - Addition of three bell-shaped curves and a constant**

### Plot of Fitted Model



**Figure 3 – Example of result of nonlinear regression (N171 direction +, June 24 to 27 2002)**

### 3.3 The probabilistic component

Counts per six minutes were available for the four stations, over a period ranging from Thursday 20<sup>th</sup> to Saturday 29<sup>th</sup> of June 2002. But to carry out predictions based on these data, it was advisable to calculate the parameters of the model on a sufficient number of similar days.

For that, four consecutive days were taken into account, from Monday 24<sup>th</sup> to Thursday 27<sup>th</sup> inclusive; Friday 28<sup>th</sup> was excluded because of the important traffic peak observed in the evening (initiating a weekend of holiday departures). Of course, even if we aim at obtaining results characterising the “weekdays”, this series is insufficient to characterise all such days in one year, especially since important seasonal variations were noted while analysing hourly data (which are available over two complete years).

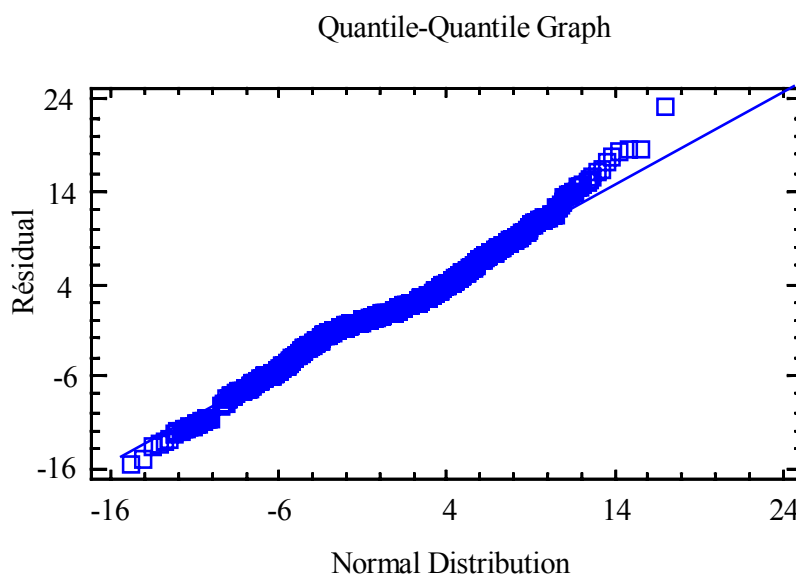
The parameters of the model were thus calculated for the four days period, and the eight counting sequences (two directions of each station); let us note that, for E60, there were several ranges of missing data, which disturb the results; for all the other stations, the data were complete.

The relevance of the model can be estimated through  $R^2$ , which is the square of the regression coefficient and expresses the share of *explained variance* (for example  $R^2=0,80$  means that 80 % of the traffic variance are explained by the model). The results range between 0,82 and 0,95; there thus remains a share of unexplained variance, which results from the addition of three phenomena: the model imperfection, differences between successive days and random variations.

Building a probabilistic model means modelling the probability of the deviations from the deterministic model: the residuals (differences between observed value and predicted value) show an average tending towards zero and a standard deviation equal to the standard error on the predictions; their study showed that they followed a normal dispersion law with an acceptable precision (see example in figure 4), which allows to base the probabilistic component of the model on this principle:

$$F(t, p) = \max(0, T(t) + S N(p))$$

Where  $T(t)$  is the deterministic component,  $S$  the standard error on predictions and  $N(p)$  the inverse normal standardised function of probability  $p$ ; the “max” function prevents the result from being negative.



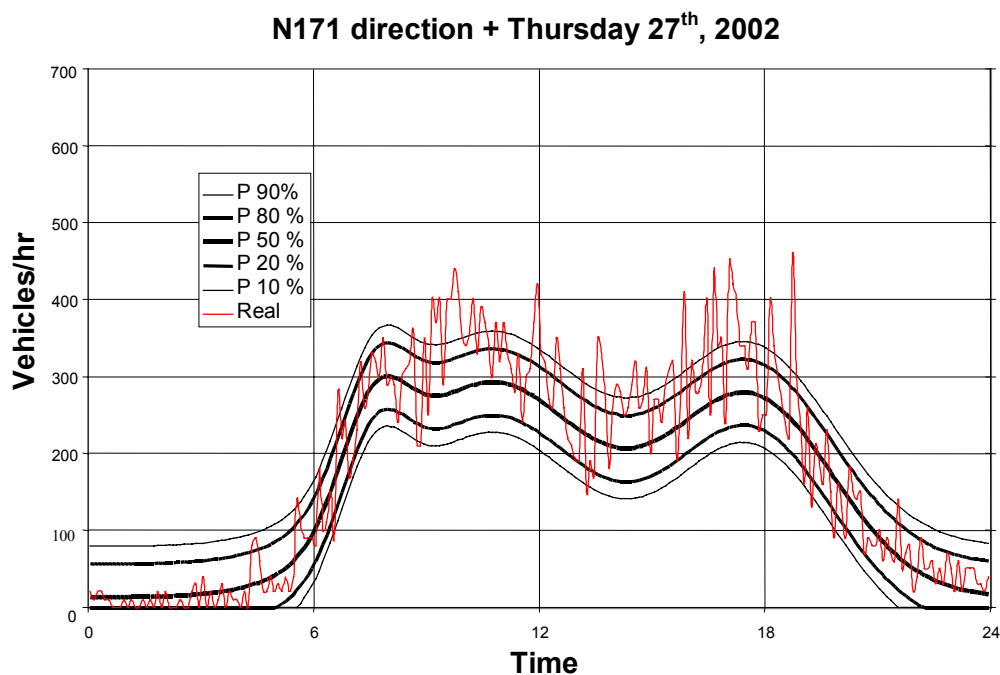
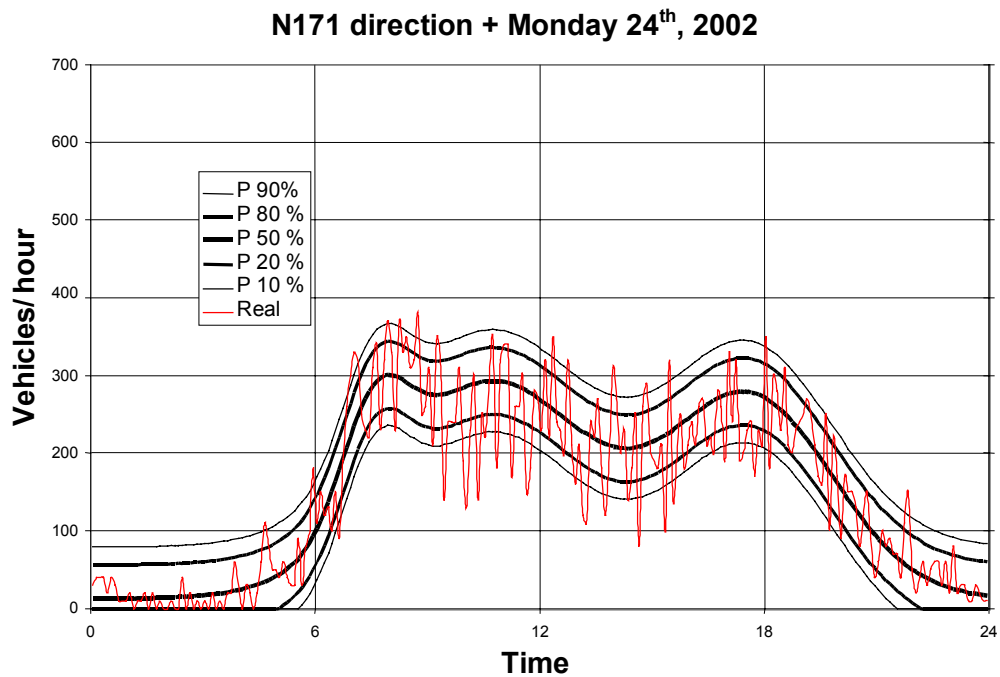
**Figure 4 – Distribution of the residuals compared to the normal law (N171 direction +, 24 to June 27 2002)**

However, two difficulties appeared:

- on one hand, a weak but significant correlation exists between the predicted value and the absolute value of the residual (in other words, the higher is predicted traffic, the higher is the uncertainty on the predicted value), but this trend is not regular enough for the model to include it;
- On the other hand, interdependence exists between successive values, which results in a positive *autocorrelation* for a time lag lower than two hours (in other words, if for  $t$  time traffic is higher than the normal, there is a significant probability for this to be true in the  $t \pm 2hr$  interval). Taking into account this kind of interaction would have required a much more complex model (of the type of what is done in weather forecasting), and, for the moment, it was given up.

Let us note finally that, if the probabilistic predictions based on a six minutes periodicity are well in agreement with real counts, calculation based on hourly counts eliminates part of the variance, and thus decreases the probabilistic term, which should then be corrected upwards, according to a coefficient which remains to be determined.

Figure 5 illustrates the application of these principles (prediction of the traffic for probabilities of 10, 20, 50, 80 and 90 % and real data).



**Figure 5 – real counts (Monday 24<sup>th</sup> and Thursday June 27<sup>th</sup> 2002) and predictions**

### 3.4 Possible simplifications

The previously defined model shows several theoretical as well as practical drawbacks:

- The concept of “normal distribution” for traffic peaks is used out of its context, and one cannot thus legitimately appeal to its properties;
- Nonlinear regression is not easy to handle (it needs a statistical software, requires a preliminary evaluation of the parameters, and does not always succeed);
- the formula comprises ten parameters to be evaluated.



To correct the first point, the concept of “Gaussian curve” is replaced by that of “bell-shaped curve”, which does not change anything with the problem, apart from the definition of the coefficients; on the other hand, the effect of “curve tails” outside the 24 hours period has proven negligible. The formula then becomes:

$$T(t) = \alpha_0 + \sum_{i=1}^{i=3} \alpha_i \exp(-\lambda_i(t - m_i)^2)$$

where  $T$ ,  $t$  and  $m_i$  keep their significance, but where  $\alpha_i$  is the traffic flow assigned to the peak, and  $\lambda_i$  a coefficient of peak sharpness (the stronger it is, the more the peak is concentrated).

The second simplification consisted in decreasing the number of coefficients; it appeared indeed that, in the large majority of the cases, there was a peak centred around midday and another around 6 PM, with coefficients  $\lambda_i$  relatively constant (around 0,12); another peak exists for the working days around 8 AM, with a higher  $\lambda_i$  (about 0,6).

If the  $\lambda_i$  and  $m_i$  are fixed at constant values, the only four coefficients  $\alpha_i$  remain to be evaluated, and this can be done through a multiple linear regression, whose use is much more convenient than that of the nonlinear regression (it is a standard function of Microsoft Excel<sup>®</sup>).

That does not change anything to the probabilistic term of the model; however this term will be slightly overestimated, given the weaker variance explained by the model.

## 4 THE TRAFFIC JAM MODEL

### 4.1 Queue calculation

Knowing the traffic flow  $Q$  at time  $t$  with a probability  $p$ , one can deduce the number of vehicles present in the queue which is formed when  $Q$  exceeds the capacity  $C$  of the road ( $Q$  and  $C$  are expressed in vehicles per hour and direction), which itself can be a probabilistic variable (but, in the present case, it will be considered as a constant).

The number of blocked vehicles results from an iterative calculation over successive periods of time (six minutes in the example), which considers that:

- if during the previous period there is no queue and if the capacity is not exceeded, then the queue length remains zero;
- in the other cases, the difference between  $Q$  and  $C$  adds algebraically to the number of vehicles present in the queue, and if the result is negative the queue disappears.

This calculation supposes of course that no diversion is possible and that the existence of the queue has no influence on the number of entering vehicles: in reality, when an important queue is formed, diversion routes are set up either in an organised way, or spontaneously, and the traffic flow is reduced. This traffic redistribution will have to be taken into account, otherwise the length of the queue may be grossly exaggerated.

Figure 6 shows an example of calculation, and the table which follows gives the quantified results. The probability with  $p$  % means here that the traffic volume has a probability of  $p$  % of being below the indicated value.

N171 direction + 24-27 June 2002 C = 250

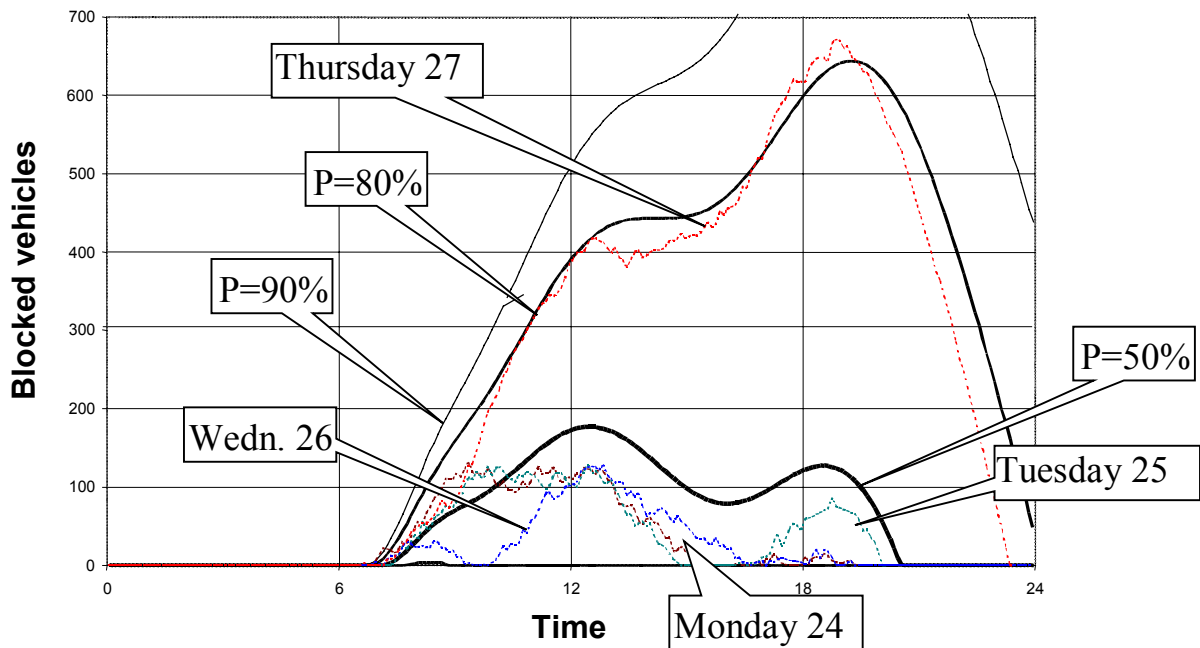


Figure 6 – Example of queue length estimation (N171 direction +, C=250 v/hr)

Table 1 - Calculated characteristics of traffic jams for two values of the residual capacity C

Data	Total Daily Traffic	C = 250 v/hr			C = 300 v/hr		
		Traffic jam duration	Maximum length (*)	Lost hours	Traffic jam duration	Maximum length (*)	Lost hours
Monday 24	3735	11h00	130	693	3h30	31	34
Tuesd. 25	3683	11h00	127	762	3h50	18	28
Wedn. 26	3701	11h45	127	488	3h20	28	27
Thursd. 27	4416	16h55	671	5923	12h05	159	847
Prob 10%	3549	0h00	0	0	0	0	0
Prob 20%	3663	1h25	4	3	0	0	0
Prob 50%	3881	13h20	177	1389	0	0	0
Prob 80%	4099	17h15	644	6557	12h30	141	820
Prob 90%	4213	>17h30	931	>9935	14h00	311	2988

(\*) maximum number of vehicles in the queue at the same time

In this example, the first three days are located at a level of probability close to 50 %, although slightly below; Thursday on the other hand draws near to 80 %.

## 4.2 Mathematical expectation of the number of lost hours

Preceding calculations allow the evaluation of queue length  $L$  at a given time  $T$ , for a given road capacity  $C$  and a given traffic flow probability  $p$ ; one can then add the successive values, in order to obtain the total of the time lost for the given period of time, made of  $N$  unit periods  $\Delta t$ :

$$H(C, p) = \Delta t \sum_{i=1}^{i=N} L_p(i\Delta t, C)$$

Calculations which follow relate to one day, with a constant capacity  $C$ . The queue length is initialised at zero for  $T = 0 \text{ hr}$ , and the calculation stops at  $T = 24 \text{ hr}$ , which means that any queue resulting from the previous day, and any transfer over the following day, will be ignored.

The following step consists in calculating average time lost per day on a high number of similar cases; this value is the mathematical expectancy  $E$ , which can result from an integral calculation of lost time  $H(C, p)$  from the probability zero to probability 1:

$$E(C) = \int_{p=0}^{p=1} H(C, p) dp$$

The integral calculation being impossible to be made in a simple way, one can use an approximation according to the formula:

$$E(C) = \sum_{i=1}^{i=n} k_i H(C, p_i)$$

where  $k_i$  is the fraction of probability allotted to  $p_i$ :

- the “commonplace” solution consists in taking  $n = 1$ ,  $k_1 = 1$ , and  $p_1 = 0.5$ , but that means ignoring completely the probabilistic aspect;
- calculation by centile takes  $n = 100$ ,  $k_i = 0.01$  and  $p_i = 0.01 i - 0.005$ ;
- $k_i$  and  $p_i$  may be optimised, in order to limit the number of iterations while improving the precision of the result.

Figure 7 shows an example of this type of calculation.

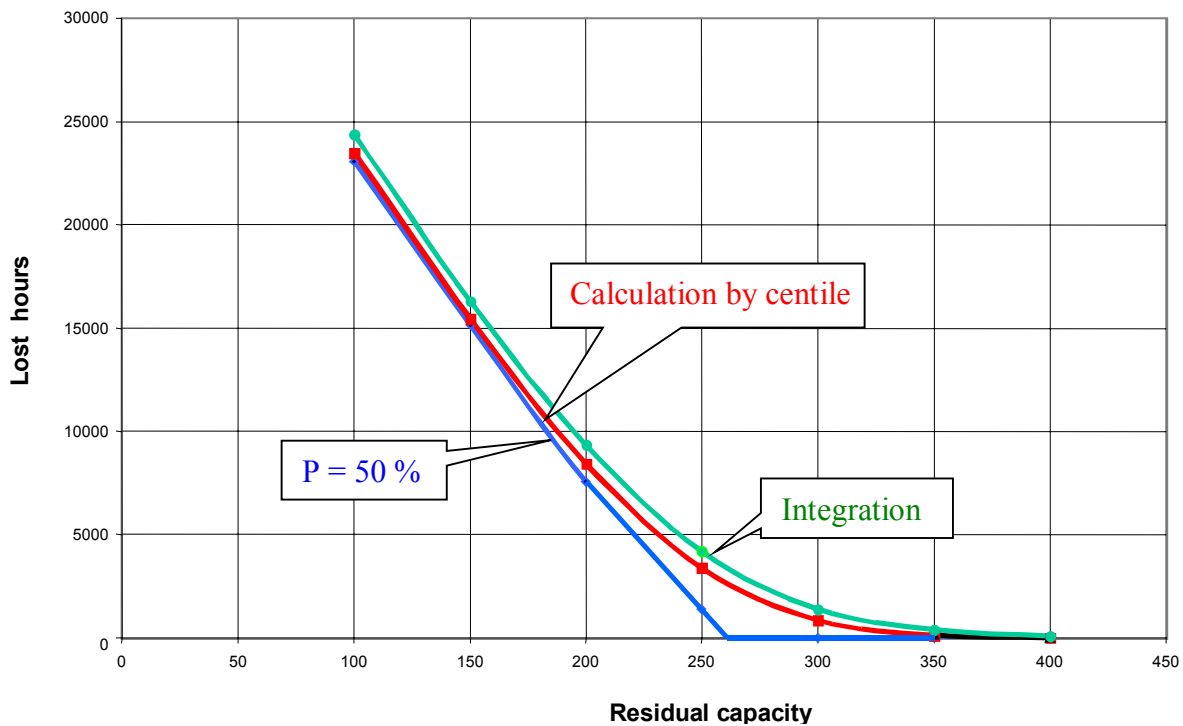


Figure 7 – Example of mathematical expectation of lost time (N171 direction +)

## 5 CONCLUSIONS

The proposed model makes it possible to calculate for any time  $t$  of the day a probability  $p$  to reach a traffic flow  $Q$ , and thus, knowing the capacity of the road  $C$  at the same time  $t$ , to determine the quantity of blocked vehicles and thus the characteristics of the queues.

This model considers the daily evolution of traffic flow as the sum of three peaks, plus a constant and a term resulting from the traffic randomness.

The coefficients can be calculated by means of a nonlinear regression based on the hourly real counts carried out on a sufficient number of days representative of the considered case; a simplified alternative consists in calculating only the height of the peaks and the basic level, the other parameters being constant, which allows the use of a multiple linear regression.

A simple model was applied to calculate the time lost in the queues for a given day probability; then the integration of the results according to the probability allows the calculation of the “mathematical expectancy” of lost time.

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