AN ANGLE-STEP-METHOD FOR DESIGNING A DRAINAGE-OPTIMIZED SURFACE FOR ROUNDABOUTS WITH SLOPE

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1. INTRODUCTION

Due to many faults in the construction of roundabouts, there exists the wish to design them correctly. The following report describes aspects of the construction and of the drainage of roundabouts with a great gradient in the longitudinal direction.

Until now there have been no international publications on the construction of these special surfaces. Therefore the solutions presented in this report are fairly theoretical, however "There is nothing more practical than a good theory" as Immanuel Kant (1724-1804), the well known German philosopher said. This report will find something about the problem leading to the angle-step-method, related guidelines, the calculated surface ("adjustable-speed surface", "Regelfläche"), the calculation method for the drainage (angle-step-method, "Winkelschrittverfahren") and finally the combination of the surface and the angle-step-method that leads to the construction of a drainage-optimized surface. Side-curves and 3D-representation of the surface-calculation and a list of unsolved questions is also found.

2. DESCRIPTION OF THE UNSOLVED PROBLEM

What makes someone think about the design of roundabouts with a great gradient in the longitudinal direction and about the drainage of such a surface? The roundabouts themselves!

It is not certain how to build the side-curves of roundabouts especially those with great gradients. The design and construction of the surface of a roundabout in relation to its drainage is not well understood and the drainage itself is meant to be an unsolved problem until now. It happens often, that trucks destroy the side-curves of roundabouts when they roll over them with one of their wheels. One of the most dangerous areas of a roundabout in relation to this destruction is the "island in the middle". At the spandrels of the roundabout at the junctions of the roads into the the ring-road areas destroyed by wheels can often be found. Usually road departments repair these areas, which unfortunately only leads to something like a patchwork rug. As an engineer you get the impression that no-one has ever designed this traffic junction in general and in detail. The problem of the drainage has not yet been solved, and it is not known where to install correctly the drains. Today the drains are often installed in the incorrect place.

These problems have to be solved, but the solution is similar solving the Gordian knot. You must create a surface in the third dimension, that can be used as a roundabout with slope, which is prepared for the largest licensed vehicle. The most important feature is the comprehensible surface drainage. The surface should also be constructed without any dividing line at the approaching roads, with a slope maximum and with an area minimum.



Figure 1: Lorry rolls over "island in the midlle" of a roundabout [1]

3. STEPS OF SOLUTION WITH AID OF KNOWN SOURCES

3.1 First step – guidelines

To solve this complicating problem it was first necessary to study some guidelines about roundabout construction (RVS 3.44 [2]from Austria, guidelines from Nordrhein-Westfalen in Germany, one guideline from Saxonia in Germany and a guideline from Lower Austria). RVS 3.42 from Austria about plane crossings and RVS 3.23 and RVS 3.8 from Austria about construction of roads were also basis of guideline-studies.

Unfortunately instead of gaining clear information, I only got confused. For example the well known element of construction "cross inclination" (q) differs from one guideline to the other. RVS 3.44 permits this slope between limits of -1.5% to -4.0%. The guideline of Nordrhein-Westfalen allows only -2.5%, the one of Saxonia has limits of -2.5% and -5.0%. In Lower Austria the cross inclination starts at -1.5% and ends at -5.0%. The other guidelines have other limits for this construction element. Unfortunately no explanation can be found as to why these limits have been fixed.

With regard to the drainage you can usually find general statements, which do not help you. Often the guidelines give a description of the correlation between the diameter of the roundabout and the width of the ring-road.

The most useful guideline is the Austrian RVS 3.44. It is possible to design a roundabout with slope – but it has to be thoroughly investigated in order to be designed.

RVS 3.44 defines the minimum of the cross inclination (q) as -1,5% (for usual use) and the maximum as -4,0%. When the longitudinal inclination (s) is not greater than 2,5% the cross inclination can be used in the given limits. But when the longitudinal slope becomes greater than 2,5% RVS 3.44 forces to incline some parts of the surface inwards. This leads to positive cross inclination, so that this slope will fall below the minimum defined by RVS 3.44.

RVS 3.44 gives only fragmented information without synopsis about roundabouts with slope.

It must be emphasized that RVS 3.44 as well as the other guidelines do not describe the roundabout-surface and roundabout-drainage completely. That is why it is not possible to find the extreme case unambiguously.



Figure 2: Roundabout with slope produced with RVS 3.44

3.2 Second step – built roundabouts

Because the guidelines did not lead to practical solutions, it was necessary to analyze some designed, constructed and built roundabouts. For the examination roundabouts from several parts of Austria were checked. Before the geometric analysis started, it was only known, that these four roundabouts were built with slope:

- Roundabout Wolfurt-Lauterach in Vorarlberg
- Roundabout Lannach Nord in Styria
- Roundabout Hallein (Burgfried) in Salzburg
- Roundabout Kirchberg am Wagram in Lower Austria

These junctions come from different Austrian regions and were designed by different engineers. Therefore different plans had to be analyzed.

To get a realistic comparison, 3D-pictures had to be made from the plans and the sequences of the longitudinal and of the cross inclination had to be determined. So it was necessary to use the angle ϕ and not the length I as comparison-parameter. That is because the axis for marking out of the ring-road differs completely. Once the axis is situated at the furthest side, once near the furthest side and twice nearer to the innerside than to the outer side of the ring-road.

The sequence of the longitudinal inclination could be determined from the contour line or from the co-ordinates, the cross inclination was often printed in the plans or had to be calculated from co-ordinates. In addition to this analysis the most important parameters were put in tabular form, so that they could be compared.

The result of this comparison is, that the roundabouts represent four completely different surfaces. They have only the slope in common. Below are more details:

The roundabout Wolfurt-Lauterach has a quite simple sequence of the slope.

The cross inclination is constant with -2.50%, the longitudinal slope varies from +1.024% to -1.024% nearly linear.

The roundabout Lannach Nord has a cross inclination, that changes between +2.50% and -2.50%. Some parts have constant +2.50% or -2.50% and others linear transitions. The longitudinal slope has a similar sequence. The maximum is +4.0%, the minimum -4.0% and between this extremes it is sometimes constant, from one extreme to the other you have linear transitions.

The sequences of the roundabout Hallein (Burgfried) are more complicated. The cross inclination follows another instruction between the extremes of +2.984% and -2.984%. The longitudinal slope has its limits at +4.542% and -4.592%.

The analysis of the Kirchberg am Wagram roundabout brings completely different slopes again. Here the cross inclination varies between +3.50% and -4.50% and the longitudinal slope has extremes of +4.80% and -6.50%.

slope	cross inclination		longitudinal inclination	
roundabout	minimum	maximum	minimum	maximum
Wolfurt-Lauterach	-2.50%	-2.50%	-1.024%	+1.024%
Lannach Nord	-2.50%	+2.50%	-4.0%	+4.0%
Hallein (Burgfried)	-2.984%	+2.984%	-4.592%	+4.542%
Kirchberg am Wagram	-4.50%	+3.50%	-6.50%	+4.80%

Table 1: The slopes of the analyzed roundabouts

The roundabout of Kirchberg am Wagram represents an extreme surface. As this roundabout has the shortest diameter of all roundabouts with 29m and the maximum longitudinal slope with -6.50% and the maximum cross inclination with -4.50% it may be assumed that the Lower Austrian roundabout is very close to the extreme surface, that has to be found.

Dispite this, there are still questions that remain unanswered:

- Why are the surfaces designed completely different?
- Who allows to make a slope constant, linear, or with an irregular sequence?
- Why is the location of the axis so varied?
- Who says, that the equipment for the drainage is always situated correctly?

It must be said, that because of different approaches different conditions predominate, that one or another engineer may have used some ideas of RVS 3.44 or that the surface had to be formed because of compulsion and constraint. But all these explanations are not justifiable for that many differences in design and construction. You also cannot find any reason for the linear transitions of the slope and no explanation in physics for that assumption.

So the analyzed surfaces can only be seen as exemplary solution-attempts. The analyzed roundabouts gave no complete description of a well-founded surface and of the drainage of this surface.



Figure 3: The four analyzed roundabouts from Austria



Figure 4: Roundabout Wolfurt-Lauterach with slope-sequences



Figure 5: Roundabout Lannach (Nord) with slope-sequences





Figure 6: Roundabout Hallein (Burgfried) with slope-sequences



Figure 7: Roundabout Kirchberg am Wagram with slope-sequences

4. STEPS OF SOLUTION WITH THE HELP OF MODELS

4.1 Third step – modelling

Due to insufficient results at all the previous investigations, I thought of a solution on my own. It should be possible to derive the surface of the roundabout and its extreme form with mathematics and physics. And it should be possible to create a method that describes the drainage. The combination of the surface and of the calculation-method for the drainage should bring a drainage-optimized surface.

The problem will be reduced to the calculation of the surface, the determination of the drainage and to the combination and optimization of the surface and the drainage. But how can you describe the surface of a road? How can you calculate the drainage of a surface? It is only possible by using mathematics and modelling. Complicated reality must be simplified.

Because there was no definition of a roundabout with slope in any guideline, I chose a definition of my own: The roundabout with slope should be defined as a special 3D-surface: As a frustum situated on an inclined plane.

When you imagine a horizontal roundabout as frustum situated on a horizontal plane you only have to rotate the roundabout around the straight line g along angle β to get the roundabout with slope. Besides this attempted solution, it was to proof that the surface is passable. That's why a very simple model of a car was needed for the following calculations.

A car itself is made from many different separate parts just as springs, wheels, shafts and so on. Sometimes it is empty, sometimes people or load has to be carried. In reality all those components work together when a car is running on the road. The moving lines of the cars become very complicated because of all these influences (swinging of the car caused by mass and springs). To get a useful and simplified model, it was necessary to give the springs an infinite stiffness. This simplification leads to a stiff axis. To achieve simplicity, only one mass of the car is taken, concentrated on the centre of gravity. The combination of the stiff axis and the mass-concentration brings the "one-axis-model". This simplification is often used in literature (e.g. [3]).



Figure 8: Definition of roundabout with slope (frustum on inclined plane)



Figure 9: The "one-axis-model" (car in curve)

4.2 Fourth step - surface calculation by dynamics

After defining all these basics, the surface of the ring-road of the roundabout with slope can be calculated with instruments of dynamics to guarantee the passability of the ring-road.

This makes it necessary to put the "one-axis model" on the frustum which is situated on an inclined plane and to place the correct forces in position. The most interesting force is the dead weight, because this force is at each position orientated vertical to the middle of the earth. Regarding the system of co-ordinates used for the calculation, the dead weight can be divided into two components. The first component is situated in the sectional plane, the second component projects out of this sectional plane. The second component can also be divided into two parts: radial and tangential. In the following calculation the tangential component is not of interest, however the radial component is attended to. This component describes the radial component of the resistance of slope.

To achieve a most pleasant drive through the roundabout with slope, it is the aim to drive the car through the roundabout with constant velocity along a circle. This would be quite easy, if there was not the radial component of the resistance of slope. If there was no force working against this resistance, the vehicle would be pushed to the middle of the roundabout at the top and would be pushed away from the middle of the roundabout at the bottom. To avoid this phenomenon, from the vehicles side an action-force (Z_r) has to be at work. But because the resistance is changing , the counteracting force also has to change in the same way.

How can this counteracting force be created as an additional lateral force? On the cars side the only possibility to realize such a force is to make different tunings of the steering. But this contradicts the circle, where always only one tuning of the steering is allowed. So it is sure, that the compensation of the resistance can not be managed from the cars side, but has to be managed from the surfaces side.

The key for this problem-solution is the angle α . It should change along angle φ this way, that the radial component of the resistance of slope always becomes zero. The result of this calculation is a surface that differs from the frustrum situated on the inclined plane. This surface is the "adjustable-speed surface" (in German: "Regelfläche"). It can be produced with a stick (= "Regal" in German), can be deduced from a frustum and can be used for driving on along a circle with constant velocity.

The mathematical description of this special surface is simple. The longitudinal inclination corresponds to those of a circle situated in an inclined plane and the cross inclination is nearly constant:

• longitudinal inclination: $s = \beta \cdot cos(\phi)$

• cross inclination: $q \approx 100 \cdot (V^2/(127,1376 \cdot R)-\mu_r)$ = constant

with: β ,s,q in [%], V in [km/h], R in [m], μ r in [-]

That the "adjustable-speed surface" has a mathematical description will be very important for the description of the drainage.



Figure 10: Determination of the surface with dynamics ("one-axis-model" and forces)



Figure 11: The "adjustable-speed surface" and its slope-sequences

5. THE KEY TO SUCCESS - ANGLE-STEP-METHOD

As illustrated, it was possible to describe the surface deduced from dynamics and to put this surface into formulas. But the problem of the drainage and its mathematical description is still not solved. This problem can be solved by using the angle-step-method (asm). The basics of the angle-step-method is the well known characteristics of water always to flow down the steepest incline. At common roads the way of the water can be

found quite simple, but when the surface has a complicating curvature you need much experience or a fast and correct method to describe the way the water follows.

The angle-step-method is an approximation method, that can describe the streamlines, even if the the surface has a complicated curvature. To calculate these streamlines it was necessary to use a mathematical trick. The rotations-symmetric area of the roundabout has to be divided into sectors. It is better to have several and very small sectors that the calculations are precise. The more sectors you have and the smaller they are the more closer to reality the result of the calculation will be. At the sectors borders the slanted inclination can be calculated from the longitudinal and from cross inclination. The slanted inclination of the border is the same as it is in the sector assigned to the border. When the sectors are very small, the difference between model and reality does not matter. Finally you can calculate the slanted inclination from both slopes and draw them down as an arrow (vector) at each border. The result of this calculation is a field of vectors representing the streaming water on the surface. A streamline can be produced like this:

Take a drop of water and let it start to flow down the surface e.g. at the boundary line of the roundabout. The drop will follow the steepest incline valid for the start-sector. When the drop has reached the border to the next sector, he will change the steepest incline, because this incline differs from the incline valid for the sector before. Then the drop follows the steepest inclination in this sector until he has reached the next border. Then it will change the steepest incline again and so on. This game can be played until the drop has reached the end.

The results are as follows:

- any desired streamline
- any desired critical streamline
- the dimension of the contribution-area for further calculations

as exact desired. It is also possible to find the right positions for the equipment of the drainage. One can see that this calculation method is powerful, as many results are received for quite different surfaces.





6. SOLUTION – SURFACE AND ANGLE-STEP METHOD COMBINED

As you see, the surface and the drainage can be expressed by formulas. So it can only be the next logical step to combine the "adjustable-speed surface" and the angle-stepmethod. This combination must lead to a drainage optimized "adjustable-speed surface". Changing the input-parameter (e.g. the velocity V or the angle β) produces changed slopes which lead to changed streamlines. So the variation of the input parameters allows the construction of the surface depending on the drainage. But you cannot only design the surface, an optimum also can be found. You can look for the steepest surface with the shortest streamlines and also for any other desired optimum.

Because this method works step by step and uses mathematical equations, you can automate these calculations and optimizations by the EDP-program EXCEL from Microsoft.

If desired, you can also determine the cartesian co-ordinates of the characteristic points and draw a 3D-picture of the surface and of the streamlines. After overlapping of the surface and streamlines, even the positions of the equipment of the drainage can be found.



Figure 13: 3D-picture: "adjustable-speed surface" (red) combined with streamline (blue)

7. REMARK – SIDE-CURVES

After designing the surface it would be possible to create the side-curves. There are also some different methods to construct them, depending on many influences. To simplify the job and to reduce the construction time of such side-curves during the investigation the side curves were made by the EDP program PLATEIA [4]. The hope was, that this program could produce 3D-side-curves. But every tractrix produced by PLATEIA was completely plane. It was also very hard and to create a realistic tractrix.

So it must be said that PLATEIA is not a useful program for producing 3D-side-curves of roundabouts with slope. By now plane side-curves have to be taken for roundabouts with

slope and the faults coming from the difference between the 3D-situation and the plane situation must be accepted.



Figure 14: Side-curves of Hirschenkreuzung in Reutte in the Tyrol

8. FURTHER COMMENTS - MODIFIED MODEL

The formulas of the

- longitudinal inclination: $s = \beta \cdot cos(\phi)$ and of the
- cross inclination: $q \approx 100 \cdot (V^2/(127, 1376 \cdot R)-\mu_r)$ = constant

with: β ,s,q in [%], V in [km/h], R in [m], μ r in [-]

are the result of the one-axis-model running on the frustum situated on the inclined plane and describe the "adjustable-speed surface". But silently another simplification was made. The height of the centre of gravity was taken with null metres above the surface. So some curves became identical, which is not realistic. To improve the model, the height of the centre of gravity was chosen greater than null metre over the surface. With this new premise all curves were calculated with the EDP program EXCEL and then analyzed again.

What was the result?

- The improved calculations produced a description of the way of the car with three curves.
- The curve of the centre of gravity remained circular.
- The car began to swing under the curve of the centre of gravity.
- The curves of the car inside the surface differ from a circle.
- The surface can be seen as a surface hanging on the curve of the centre of gravity.
- The more details means more parameters which produces a more complicating design.
- The approaches have to be checked very carefully and to be designed in detail.
- The angle-step-method kept on working without any trouble.

This result means that the leading-curve for the tractrix cannot be deduced from a circle and will probably not be circular itself.

The improved model shows the problem of the approaches better than the simple model. You can see the cracks coming from the "adjustable-speed surface" and the inclined plane. The simplest solution of a roundabout with slope without any crack to the approaches will probably be an inclined plane. This plane will be the best for the approaches but not for the drainage and the comfort of driving. Because at the inclined plane the water would flow to the middle and it would not be possible to drive a circle with constant velocity (The "adjustable-speed surface" cannot be reduced to an inclined plane). So the simplest solution for approaches without cracks works against the best solution for driving and drainage.



Figure 15: 3D-picture: unfinished "adjustable-speed surface" (red) made with modified model



Figure 16: Details: "adjustable-speed surface" (red), streamline (blue), car (green, blue, purple)



Figure 17: Details of 3D-streamline calculated by angle-step-method

9. THE RESULT

Finally it can be said that it is now possible to describe a surface of a roundabout with slope, that can be used for driving on. This surface is the "adjustable-speed surface". With the angle-step-method now it is possible to descibe the drainage of such a surface and to find out critical points of the drainage.

The combination of the "adjustable-speed surface" and the angle step method allows to optimize the surface depending on the drainage. Open is the question of the side curves produced by a tractrix. It must even be suspected that this problem is not purely a geometrical but also a dynamic problem. The question of the approaches without cracks is still not solved but in discussion.

What can be learned for other disciplines of engineering? Modeling and simplification often leads to results. But you have to try to go unusual ways. The angle step method is so powerful, that it works for roads as well as for the substructure-plane of railroads, for roofs of houses and so on. So the Gordian knot is almost solved.

10. SUMMARY

Roundabouts are becoming increasingly popular in Europe. Even hilly countries like Austria discover this form of junction. So roundabouts with great slope have to be built in hilly regions. But roundabouts are still not perfect today and especially those with a great slope have many faults in their construction. Because of these faults, there exists the wish to design roundabouts correctly. The design of many elements of the roundabouts remains unknown today. So it is uncertain how to build the side-curves of roundabouts especially of those with a great gradient. The design of the surface of a roundabout in relation to its drainage is not so well understood and the drainage itself is meant to be an unsolved problem until now.

It happens often that trucks roll over the side-curves of roundabouts with one of their wheels and then they destroy the sides of the roundabouts. This destruction can be seen in the middle-area and near the approaches. Road-departments repair those destructions, but produce "patchwork-roads".

To avoid such situations it is necessary to solve the problem like this. It should be created a surface in the third dimension, that can be used as a roundabout with slope, prepared for the largest licensed vehicle, with the most important characteristic: the comprehensible drainage. Furthermore the roundabout should be built without any dividing line at the approaching roads, with maximum of slope and on a minimum of area.

This problem, similar to opening the Gordian knot, led to many possible solutions.

The first attempt, studying five different guidelines concerning roundabouts or their construction did not bring success but rather confusion. Even the well known element of construction "cross inclination" differs from one guideline to the other between –1.50% and –5.00%. Unfortunately you can not find any explanation as to why these limits have been fixed. Regarding the drainage you can usually find general statements, which are unhelpful. Often the guidelines gave a description of the correlation between the diameter of the roundabout and the width of the ring-road. The best guideline was the Austrian RVS 3.44 about roundabouts, because it allowed the construction of roundabouts with slope. Finally it has to be emphasized that the RVS 3.44 as well as the other guidelines do not describe the roundabout-surface and roundabout-drainage completely. That is why it is not possible to find the extreme case unambiguously.

The second attempt was the analysis of four ready built roundabouts in Austria, which have a great slope. From these roundabouts it was only known, that they have a slope. The result of this investigation were four completely different surfaces with different sequences of the cross inclination and the longitudinal inclination.

So the analyzed surfaces can only be seen as exemplary attempts of solving the problem. The analyzed roundabouts give no complete description of a well-founded surface and of the drainage of this surface.

Because there were only insufficient results with all the previous investigations, I thought of a solution on my own. With a very reduced model it should be possible to solve the problem. But before modelling, it was necessary to make some definitions.

So a roundabout with slope was defined as a special 3D-surface, that means as a frustum situated on an inclined plane. Beside this attempt of solution for the surface, it was to proof of that the surface is passable. That is why a very simple model of the car was needed for the following calculation.

The simplification of the car lead to a stiff axis-model. And all the different possible masses of the car were reduced to only one mass, concentrated on the centre of gravity. The combination of the stiff axis and the mass-concentration brings the "one-axis-model".

After defining all these basics, the surface of the ring-road of the roundabout with slope can be calculated with instruments of dynamics to guarantee the passability of the ring-road.

This makes it necessary to put the "one-axis model" on the frustum situated on an inclined plane and to place the correct forces in position. Then the car-model could run on the frustum situated on an inclined plane. But one force must be compensated, to achieve the car running through the roundabout along a circle with constant speed. Because this could only be realized by giving the roundabout a special form, the surface was defined. The solution-surface should have constant cross inclination and a longitudinal inclination

similar to the longitudinal inclination of a circle situated on an inclined plane. The "adjustable-speed surface" was created.

To realize the correct drainage it was necessary to develop a method, that could describe the streamlines of the water on the surface. The key was the angle-step-method, that can describe the way of the water flowing down the steepest incline step by step and as precisely as needed. My idea was to approximate the surface by sectoral planes. Along these planes the slanted inclination should be constant. At the borders of the section the direction of the water has to change its way because the slanted inclination changes. So the streamline originates step by step, sector by sector.

The results were as follows:

- any desired streamline
- any desired critical streamline
- the dimension of the contribution-area for further calculations

as exact as you want.

The method can be expressed by mathematical formulas, which makes it possible to combine the "adjustable-speed surface" and the angle-step-method.

Using the program EXCEL the combination can be calculated and optimized. This leads to a drainage-optimized "adjustable-speed surface" for roundabouts with slope. The problem of the side-curves could not be solved because the EDP-program PLATEIA only produced plane tractrix-curves. Here faults because of the difference between plane and slope have to be accepted.

A higher tuned model produces some interesting results.

- The curve of the centre of gravity remained circular.
- The curves of the car inside the surface differ from a circle.
- The angle-step-method kept on working without any trouble.

Finally it can be said that it is now possible to describe a passable surface of a roundabout with slope. This surface is the "adjustable-speed surface". With the angle-step-method now it is possible to descibe the drainage of such a surface and to find out critical points of the drainage. The combination of the "adjustable-speed surface" and the angle step method allows the optimisation of the surface depending on the drainage. So the Gordian-knot-like problem has a solution.

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